

CEM PRE-COURSE MATH PREP

Barney Capehart, PhD & Updated by Eric Woodroof, PhD

Thank you for taking interest in the Certified Energy Manager Training Program. By attaining this professional certification, you will be joining an elite group of professionals serving industry, business and government throughout the United States and in over 100 countries abroad. These high-achieving individuals comprise a "Who's Who" in the energy management field.

The CEM is a well-respected certification, partly due to the amount of material that is presented and the breadth of subjects (Motors, Lighting, Boilers, Economics, Procurement, etc.). There are many qualitative concepts presented in the course that will help you save energy in “real world” applications.

However, attendees should also have some basic algebra skills to keep up with the pace of the course. To give you the best opportunity during the class and exam, we recommend you review Items below prior to the course and make sure you can handle the types of questions in the CEM Prep Quiz.

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1. Things to do prior to the Course

Prior to the course it is very important for everyone to do:

- A. Get something to tab your workbook.** Narrow “Post-it Notes” or something similar is effective at marking the workbook pages so that you can quickly reference them during the exam. (THIS IS PROVIDED AT “IN-PERSON” & “VIRTUAL” COURSES).
- B. Get a calculator.** A basic calculator that can do $+/-/*\sqrt{}$ will be sufficient. While there are many great calculator “apps”, devices that have a camera or communication ability will not be allowed on your desk during the exam, so you need a separate calculator. *Although not necessary, you can use a scientific or financial calculator, just practice using it before you attend class.*
- C. Review the CEM Preparatory Quiz.** We recognize that many course participants do not do algebra every day. However, we will be solving complex equations during the class, so please take some time and review the problems and be comfortable solving them.
- D. Dress Code:** Business casual, but we do recommend bringing a jacket or sweater, because some venues can have rooms that are kept colder than you might find desirable.

2. Pre-Course Resources that may be helpful

If you are not familiar with fundamental energy-related concepts, you can watch FREE recorded webinars (less than 10 min each) to prepare in advance. If you need to review “Time Value of Money” concepts like NPV and LCC, please be sure to watch videos “A” and “B”. The videos can be found here:

https://www.youtube.com/playlist?list=PLkJJrXF_HpGWMk55HGTJ7zyOxmFI0unQ4

Or Search YouTube: “AEE CEM Basics Series”

3. CEM PREP QUIZ

If you can successfully complete the questions in 25-30 minutes, and get most of the correct answers, you are probably going to be OK to start the class.

If you cannot successfully complete the questions (or miss many of them) in 30-40 minutes, you need to improve proficiency. *Now is the time to focus on improving your basic math skills. In this case, you may choose to study The Basic Math Skills Review (following the quiz below).*

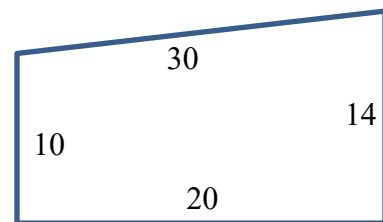
OK... lets see how you do:

A. Basic Calculator Skills

1. Find $500 \times 0.746 \times 0.8/0.93 =$ _____
2. Find $(325 \times 10^7 \text{ Btu} + 210 \times 10^5 \text{ Btu})/100,000 \text{ ft}^2 =$ _____ Btu/ft²
3. Find $500 \times 10^6 \times 3500 \times 24 \times 4/0.75 =$ _____
4. Find $50 \times (350/500)^3 =$ _____
5. Find $50 \times \sqrt{(350/500)} =$ _____

B. Basic Algebra and Equation Manipulation Skills

6. What is the perimeter of the following room?
Perimeter = _____ feet



7. A formula we will use in electrical analysis is:

$$P_{in} = \sqrt{3} \times kV \times A \times PF$$

Just working with these parameters, what is the solution to this formula for kV?

kV = _____

8. Another formula we will use in HVAC analysis is

$$q = U \times A \times \Delta T$$

How could this be re-written as shown below?

$$U = \underline{\hspace{2cm}}$$

9. What is the volume of a cylindrical heating oil tank that is 100 feet in diameter, and 10 feet tall? Use $\pi = 3.1416$. Volume = $\underline{\hspace{2cm}}$ ft³

C. Basic Concepts Needed

10. Is kW a measure of energy or a measure of power? $\underline{\hspace{2cm}}$

11. A facility electric bill was \$25,000 last year. John put in a new high efficiency air conditioning system and his electric bill this year was \$21,500. What percent savings in his electric bill did he achieve? $\underline{\hspace{2cm}}$ %

12. Note: The definition of efficiency is the ratio of the system output in Btu divided by the input in Btu. Our facility has a space heater that is 75% efficient. If we had to buy 20 million Btu of fuel for our heater, how many Btu of heat did we deliver to the heated space in our building? $\underline{\hspace{2cm}}$ million Btu

13. Our facility has an 80% efficient hot water heater, and it provided us with 10 million Btu of hot water last month. How much natural gas fuel did we have to buy in units of Btu last month? $\underline{\hspace{2cm}}$ million Btu

14. A facility uses 2.17×10^8 Btu of electrical energy and 8.13×10^6 Btu of fuel energy in a year. What is the total Btu the facility used in that year? $\underline{\hspace{2cm}}$ million Btu

15. The ABC Company used 3.5 million kWh in 2011. Their cost was \$0.12 per kWh. How much did the ABC company pay for electric energy in 2011? \$ $\underline{\hspace{2cm}}$

16. Heating oil contains 140,000 Btu of energy per gallon of oil. Natural gas contains 1000 Btu of energy per cubic foot of gas. How many cubic feet of natural gas is required to provide the same energy as 500 gallons of heating oil? _____ft³

17. Are the two expressions shown below algebraically equivalent?

 kW/ton and kW·h/ton·h

Yes

No

18. A barrel of oil contains about 5,100,000 Btu of energy. One kWh of electrical energy contains 3412 Btu of energy. How many kWh are contained in 16 barrels of oil? _____kWh

19. Heating oil costs \$3.00 per gallon, and each gallon of oil contains 140,000 Btu of energy. The oil is burned in a space heater with efficiency 75%. What is the cost of providing one million Btu of heat from the heater?
\$ _____

Answers

1. 320.86
2. 32,710 Btu/ft²
3. 2.24×10^{14}
4. 17.15
5. 41.83
6. 74 feet
7. $kV = P_{in}/(\sqrt{3} \times A \times PF)$
8. $U = q/(A \times \Delta T)$
9. 78,540 ft³
10. Power
11. 14%
12. 15 million Btu
13. 12.5 million Btu
14. 225.13 million Btu
15. \$420,000
16. 70,000 ft³
17. Yes
18. 23,915.6 kWh
19. \$28.57

Resource: Basic Math Skills Review (if you didn't do well on the quiz)

This section is broken into the following sections:

Order of Operations / Hierarchy

Calculators

Expressions

Conversion Factors for Unit Conversions

Unit Conversions

Equations

Order of Operations / Hierarchy:

The order of operations or hierarchy is the order in which a mathematical expression or equation should be conducted. The hierarchy of operations is as follows:

1. Operations inside grouping symbols
2. Exponents
3. Multiplication and/or Division in the order of left to right
4. Addition and/or subtraction in the order of left to right

Examples:

1. $7 + 3 \times 4 + 6$

For this expression, the first operation must be the 3×4

$$7 + 12 + 6$$

The next order would be the addition, in order from left to right

25

2. $6 \times 7^2 \times 5 + 14 \times .9^2$

For this expression, the first operation must be the exponents

$$6 \times 49 \times 5 + 14 \times .81$$

The next order would be the multiplication, in order from left to right

$$1,470 + 11.34$$

The next order would be the addition, in order from left to right

$$1,481.34$$

3. $133 \times (.06^2 + .125 \times 420) + 12^3$

For this expression, the first operation must be the exponents (we must reduce these to fully do the operation within the grouping symbol)

$$133 \times (.0036 + .125 \times 420) + 1,728$$

The next order would be the multiplication within the grouping symbol

$$133 \times (.0036 + 52.5) + 1,728$$

The next order would be the addition within the grouping symbol

$$133 \times (52.5036) + 1,728$$

The next order would be the multiplication

$$6,982.9788 + 1,728$$

The next order would be the addition

$$8,710.9788$$

4. $1,786 + (7 \times 3 / 3 \times 7)^2 \{14 + 24 - 14 + 24\} + [80 / 100]^3$

For this expression, the first operation must be the operations within the grouping symbols. For the first grouping symbol, multiplication in the order from left to right; the second grouping symbols is addition and subtraction in order from left to right; the third grouping symbol is the performance of the division.

$$1,786 + (49)^2\{48\} + [.80]^3$$

The next order would be the exponents

$$1,786 + 2,401 \times 48 + .5120$$

The next order would be the multiplication

$$1,786 + 115,248 + .5120$$

The next order would be addition, in order from left to right

$$117,034.512$$

Expressions:

An expression is a mathematical phrase that is a combination of one or more numbers, variables, constants combined by one or more operational symbols. A numerical expression is a mathematical phrase that involves only one or more numbers combined by one or more operational symbols.

Here are five examples of expressions:

$$1,786 + 115,248 + .5120$$

$$140,000 \text{ gal} + 235,000 \text{ gal}$$

$$6X + 4Y + 2Z + 6U$$

$$\frac{2.065 \times 10^6 \frac{\text{Btu}}{\text{annually}} + 14.5 \times 10^6 \frac{\text{Btu}}{\text{annually}}}{175,000 \text{ ft}^2}$$

All of the above items are expressions, even the last one which would be how to calculate an energy use index (EUI). They can be reduced or simplified. One key to understanding the difference between an expression and an equation is **there IS NO Equal sign** in an expression. It is not equal to anything, Yes – it can be reduced or simplified or converted to another item but it is not equal to anything.

Unit Conversions:

As a CEM, it is very important that we can convert from one like energy related unit to another. Since we have several different basic energy units and many different energy unit multipliers, we constantly face the task of converting from one set of energy units to another. These conversions can be handled in a very systematic approach that can be applied to basic conversions, and to more complex conversions and calculations in the future.

The principle of performing unit conversions is simply to carry out algebraically correct multiplications and divisions using correct units at each step, starting with the given piece of information and transforming it into the desired units with the use of one or more conversion factors. If we ever perform one of these basic unit conversion calculations, and find that we have different units on the left and the right. We do not have the correct answer in terms of the desired units. This method is given the colloquial name Railroad Track Method, because the vertical separation lines remind us of railroad tracks. Before we start, here are the conversion factors that we will use in class and because you're in the energy management field – You'll use these extensively in your work!

Conversion factors for Unit Conversions:

1 kWh.....	3,412 Btu
1 ft ³ natural gas.....	1,000 Btu
1 Ccf natural gas.....	100 ft ³ natural gas
1 Mcf natural gas.....	1,000 ft ³ natural gas
1 therm natural gas.....	100,000 Btu
1 barrel crude oil.....	5,100,000 Btu
1 ton coal.....	25,000,000 Btu
1 gallon gasoline.....	125,000 Btu
1 gallon ethanol.....	84,400 Btu
1 gallon #2 fuel oil.....	140,000 Btu
1 gallon LP gas.....	95,000 Btu
1 HP.....	746 Watts
1 Mbtu.....	1,000 Btu
1 MMBtu.....	1,000,000 Btu
1 Decatherm.....	1,000,000 Btu
1 therm	100,000 Btu
1 Quad.....	1x10 ¹⁵ Btu
1 MW.....	1,000,000 Watts

Let's walk through a couple of examples:

1. How many quarts are in 10 Gallons of Milk?

(We realize that you probably have the answer already but let's see how to use the process. First draw a common fraction bar (the Track)

Next we will always write in the top left, what we are given. In this example, we are given 10 gallons of milk

10 Gallons

Next we draw a separation line

10 gallons |

Now we need to insert a conversion factor that will convert gallons to quarts. The conversion factor is 4 quarts in 1 gallon. We will need the units of gallons to cancel – so the 1 gallon must go on the bottom.

10 gallons | 4 quarts

| 1 gallon

Next we will cross cancel the units of Gallons, and we're left with

10

| 4 quarts

| 1

At this point, we simply multiply, 10 times 4 which will equal 40, then bring the units, so in 10 gallons of milk, there are 40 quarts.

2. Next, let's walk through a little more complex conversion, but again follow the same process. For this example we want to know how many therms are contained in 4,750 gallons of #2 fuel oil?

First draw a common fraction bar (the Track)

Next we will always write in the top left, what we are given. In this example, we are given 4750 gallons of #2 fuel oil

4,750 Gallons

Next we draw a separation line

4750 gallons |

|

Now we need to use a conversion factor that will convert gallons to therms, but we see from the conversion factor list, there isn't one, so we need to select a middle step by converting the gallons to Btu's, and then converting the Btu's to therms. In this first step we will need the units of gallons to cancel – so the 1 gallon must go on the bottom

4750 gallons | *140,000 Btu*

| *1 gallon*

Next we will cross cancel the units of Gallons, and we're left with

4750 | *140,000 Btu*

| *1*

Next we will enter the next conversion, from Btu's to Therms

4750 | *140,000 Btu* | *1 Therm*

| *1* | *100,000 Btu*

Next we will cross cancel the units of Btu's, and we're left with

4750	140,000	1 Therm
	1	100,000

At this point, we simply multiply and divide; 4,750 times 140,000 and divide this by 100,000. This gives us an answer of 6,650 Therms of energy in 4,750 gallons of #2 fuel oil.

Now try some examples for yourself. (answer key is right after the set

1. How many kWh are there in 100,000 cubic feet of natural gas?
2. How many Btu's are there in 2.6 Million kWh?
3. How many MMBtu are there in 450,000 therms of natural gas?
4. How many MMBtu are there in 500,000 gallons of #2 fuel oil?
5. How many gallons of gasoline (in Btu content) would be the equivalent to having 5,000 gallons of ethanol?

Answers:

1. How many kWh are there in 100,000 cubic feet of natural gas?
29,308.32 kWh
2. How many Btu's are there in 2.6 Million kWh?
8,871,200,000 Btu
3. How many MMBtu are there in 450,000 therms of natural gas?
45,000 MMBtu
4. How many MMBtu are there in 500,000 gallons of #2 fuel oil?
70,000 MMBtu
5. How many gallons of gasoline (in Btu content) would be the equivalent to having 5,000 gallons of ethanol?
3,376 gallons

Equations:

An equation is a mathematical statement in which two expressions are equal to each other. Here are the formal steps to solving equations:

1. Reduce all fractions and combine like terms whenever possible
2. Multiply both side of the equation by the lowest common denominator
3. Isolate all terms that contain the variable to one side of the equation
4. Factor out the variable that you wish to solve for

- 5. Divide both sides of the equation by the expression the factor came from**
- 6. Reduce the solution if possible**
- 7. Check your work**

Now let's walk through an example: Solve for the following equation for X?

$$\frac{24x}{6} = \frac{20x}{100} + 15y - 12z$$

In this first step we will reduce all fractions

$$4x = .2x + 15y - 12z$$

In the next step we will get all of the terms that contain the variable x to one side of the equation. We do this by adding or subtracting. If we move one term to the other side of the equation we need to change its sign. Recognize this is a shortcut, if we showed it the full way in this example – we would show that we would be subtracting .2x from both sides of the equation. On the right side when we do this subtraction it would equal zero)

$$4x - .2x = 15y - 12z$$

In this step we can combine like terms

$$3.8x = 15y - 12z$$

To show how we factor the unknown out we'll factor the variable out

$$x(3.8) = 15y - 12z$$

Then, we'll divide both sides of the equation by the expression in the parenthesis

$$\frac{x(3.8)}{(3.8)} = \frac{15y - 12z}{(3.8)}$$

The 3.8 on the left side will cancel each other out, so the answer to this problem is because it cannot be reduce further

$$x = \frac{15y - 12z}{3.8}$$

Remember the process to solve equations and recognize that the more practice you do the better and faster you will become at solving them. Here are a couple of shortcuts that can help you.

Since it's an equation, what you do to one side you have to do to the other to keep it equal. When we look at the final results of this balancing we can move items to the other side of the equation based upon the following:

1. If the term that we are moving is in the numerator we can move it to the other side of equation, we can move it into the denominator by simply changing the sign of it. We can also move it to the other side by moving it to the denominator.
2. If the term that we are moving is in the denominator we can move it to the other side of equation, we can move it to other side if we move it to the numerator.